AIP Advances

Effect of magnetic field tapering on whistler pumped free electron laser

Cite as: AIP Advances **9**, 025207 (2019); https://doi.org/10.1063/1.5064751
Submitted: 08 October 2018 . Accepted: 01 February 2019 . Published Online: 12 February 2019

Manish Kumar

Effect of magnetic field tapering on whistler pumped free electron laser

Cite as: AIP Advances 9, 025207 (2019); doi: 10.1063/1.5064751
Submitted: 8 October 2018 • Accepted: 1 February 2019 •
Published Online: 12 February 2019

Manish Kumar[a)]

AFFILIATIONS

Department of Electrical Engineering, Indian Institute of Technology BHU, Varanasi 221005, India

[a)]Email id: mkumar.eee@itbhu.ac.in

ABSTRACT

An analytical formalism of whistler pumped free electron laser (FEL) with tapered magnetic field is developed. The tapering raises the efficiency of the device to 7% for typical parameters. The frequency and power of the FEL can be controlled by tuning the plasma density and/or magnetic field also by increasing the energy of electron beam. The angle of injection of electron beam is very crucial for higher efficiency of FEL. Presence of plasma ensures the space charge and current neutralization and larger power handling capacity of the device.

I. INTRODUCTION

The conventional terahertz sources offered very little energies of the order of 10–100 μJ per pulse for the beat excitation of THz radiation using Gaussian laser beams co-propagating along the direction of ambient magnetic field in a rippled density plasma channel.[1] The measured value of the coherence properties of SACLA (Spring-8 Angstrom Compact Free Electron Laser) on the basis of a single shot at an X-ray are 4 – 30 keV, time of 0.1 fs and a pulse duration of 5.2 fs respectively.[2] Vogt, Faatz, Feldhaus, Honkavaara, Schreiber, Treusch[3] have examined the free-electron laser FLASH at DESY routinely produces up to several thousand photon pulses per second with 44 nm down to 4.2 nm wavelength at up to 500 μJ pulse energies. Yoshida, Munemoto, Kimura, Sano, Yamaguchi, and Asakawa[4] have developed a compact Double-Slab Type Cherenkov Free-Electron Laser Resonator for the resonant frequency and the small signal gain of 6% with a 50 kV/3.6 mA electron beam and saturation power was estimated to be 90 mW. Hasanbeigi have carried out experimental work on Growth rate enhancement of free-electron laser by two consecutive wigglers with axial magnetic field more than the growth rate of a conventional FEL.[5]

S. H. Gold et al. experimentally demonstrated a high power FEL amplifier (FELA) using relativistic electron beams (REBs) at 35GHz for 1.2 dB/cm growth rates and experimental efficiency > 3% with 50 dB gain. They have also examined an effect of tapering on axial magnetic field to enhance the efficiency and power of the device that indicate the production of > 75 MW at 75 GHz with experimental efficiency of 6%.[6,7] Orzechowski et al.[8] examined the operation of a FELA at 35GHz with a peak output power of 180MW and powered by 3.6 MeV, 850 A of electron beams and find out an extraction efficiency of 6% with operating bandwidth of approximately10% to 1.4m wiggler lengths. Sharma et al.[9,10] examined the operation of the electromagnetic and electrostatic wiggler with whistler pumped free electron laser for the higher frequencies. Pant and Tripathi[11,12] have proposed a nonlocal theory and studied their operation in whistler mode, also examined the operating possibilities using a strong axial guided magnetic field and a static magnetic wiggler. Presence of plasma ensures the space charge and current neutralization and larger power handling capacity of the device. Magnetized plasma is necessary for using whistler as an undulator for a FEL radiation source.

In this paper, we study an effect of linear tapering of the magnetic field on whistler pumped free electron laser and proposed to use a gyrotron source of an electromagnetic wiggler wave that offers the possibility of generating short radiation wavelengths using tapered magnetic field in presence of the uniform background plasma medium. The simulation is closely related to the Sharma et al.[9,10] with key difference from the experiment of S. H. Gold et al[6,7] is using gyrotron source of an electromagnetic wiggler wave with a desire to improve the efficiency > 6%. In section II, analytical formalism has been done. The results are briefly discussed in section III.

II. INTERACTION REGION OF FEL

Consider the interaction region, $0 < z < L_s$ of a free electron laser from Fig. 1, where L_s is the length of interaction region. The calculations are done for 1D calculations and for uniform electron beam. It comprises a plasma of electron density n_0 immersed in a static magnetic field $B_s\hat{z}$. The magnetic field has a linear taper,

$$B_s = B_{os}(1 - z/L_s) \tag{1}$$

A circularly polarized whistler wave propagates through the plasma as a wiggler along $-\hat{z}$. In the WKB approximation its electric field can be written as,

$$\vec{E}_o = A_o(\hat{x} + i\hat{y})e^{-i(\omega_o t + \int k_o dz)} \tag{2}$$

where, $k_o = \frac{\omega_o}{c}(1 - \frac{\omega_p^2}{\omega_o(\omega_o - \omega_c)})^{1/2}$, $A_o = \frac{A_{oo}}{(ck_o/\omega_o)^{1/2}}$, $\omega_c = \omega_{co}(1 - z/Ls)$, $\omega_{co} = eB_{os}/m$, $\omega_p = (n_0 e^2/m\varepsilon_o)^{1/2}$, $-e$ and m are the electronic charge and rest mass, ε_o is the absolute permittivity of free space, c is the speed of light in vacuum. For we seek k_o to be an increasing function of z hence a negative taper in magnetic field is considered. A relativistic electron beam of density n_{ob} and velocity $v_{ob}^o\hat{z}$ propagates through the plasma. It acquires an oscillatory velocity due to the whistler. Solving the relativistic equation of motion, one obtains

$$\vec{v}_{ob} = \frac{e\vec{E}_o(1 + k_o v_{ob}^o/\omega_o)}{mi\gamma_o^o(\omega_o + k_o v_{ob}^o - \omega_c/\gamma_o^o)}$$

where

$$\gamma_o^o = (1 - v_{ob}^{o}/c^2)^{-1/2} \tag{3}$$

The plasma electrons also acquire drift velocity

$$\vec{v}_o = \frac{e\vec{E}_o}{mi(\omega_o - \omega_c)} \tag{4}$$

We launch a circularly polarized FEL radiation through the $z = 0$ end with electric field

$$\vec{E}_1 = A_1(\hat{x} + i\hat{y})e^{-i(\omega_1 t - k_1 z)} \tag{5}$$

where $k_1 = \omega/c$, $\omega >> \omega_p, \omega_c$. It imparts oscillatory velocities to beam and plasma electrons

$$\vec{v}_{1b} = \frac{e\vec{E}_1}{mi\omega_1\gamma_o^o}, \vec{v}_1 = \frac{e\vec{E}_1}{mi\omega_1}$$

The whistler and FEL beat to exert a ponderomotive force on them at ω, k where

$$\omega = \omega_1 - \omega_o, k = k_o + k_1$$

Then the phase matching condition to be satisfied is,

$$v_{ob}^o \approx \frac{\omega_1 - \omega_o}{k_1 + k_o} \tag{6}$$

After rearranging of the above, we have

$$\omega_1 \approx 2\gamma_o^{o2}\omega_o(1 + \frac{v_{ob}^o}{c}\varepsilon^{1/2}),$$

where

$$\varepsilon = (1 - \frac{\omega_p^2}{\omega_o(\omega_o - \omega_c)}) \tag{7}$$

The phase-space dynamics of a relativistic electron beam in the presence of the ponderomotive force

$$\begin{aligned}
\vec{F}_{pb} &= e\nabla\varphi_{pb} = -\frac{e}{2}\vec{v}_{ob}^* \times \vec{B}_1 - \frac{e}{2}\vec{v}_{1b} \times \vec{B}_o^*,\\
\vec{F}_p &= e\nabla\varphi_p = -\frac{e}{2}\vec{v}_o^* \times \vec{B}_1 - \frac{e}{2}\vec{v}_1 \times \vec{B}_o^*,\\
\vec{B}_o &= -\frac{k_o}{\omega_o}\hat{z} \times \vec{E}_o, \vec{B}_1 = \frac{k_1}{\omega_1}\hat{z} \times \vec{E}_1,\\
\varphi_{pb} &= -\frac{eA_o^*A_1 e^{-i\psi}}{m\omega_o\omega_1\gamma_o^o(k_o + k_1)}(k_o + \frac{(\omega_o + k_o v_{ob}^o)k_1}{(\omega_o + k_o v_{ob}^o - \omega_c/\gamma_o^o)}),\\
\varphi_p &= -\frac{eA_o^*A_1 e^{-i\psi}}{m\omega_o\omega_1(k_o + k_1)}(k_o + \frac{\omega_o k_1}{\omega_o - \omega_c}),\\
\psi &= \omega t - \int(k_o + k_1)dz
\end{aligned} \tag{8}$$

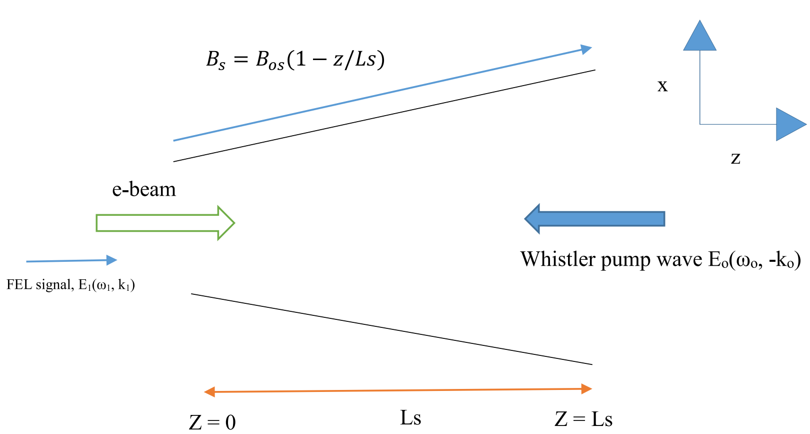


FIG. 1. Schematic of Tapered Magnetic Field FEL.

TABLE I. Comparison of efficiency and propagation constant of FEL generation by varying injection angle of electron beam.

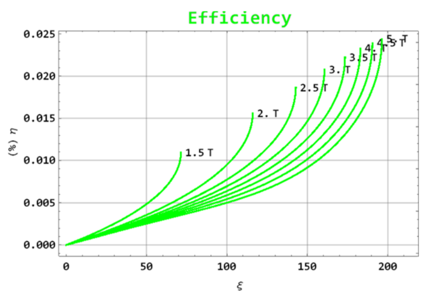
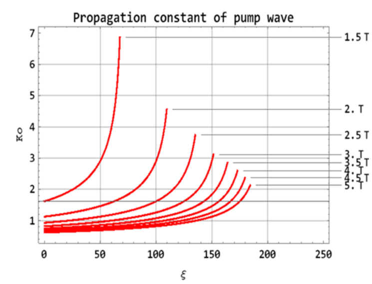
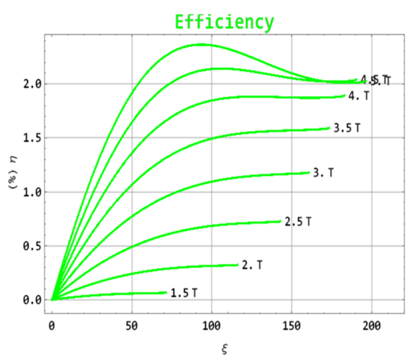
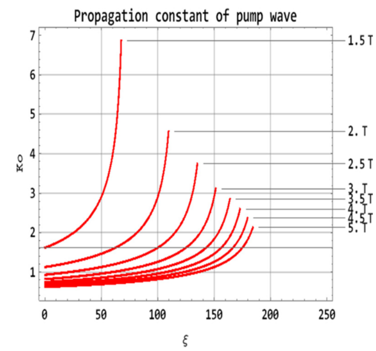
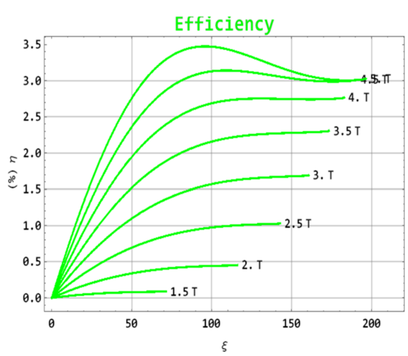
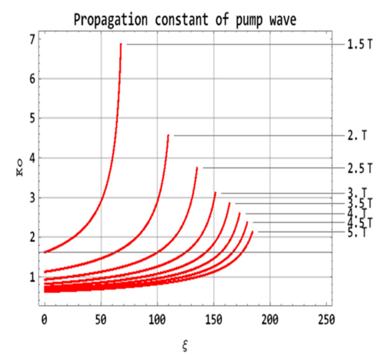
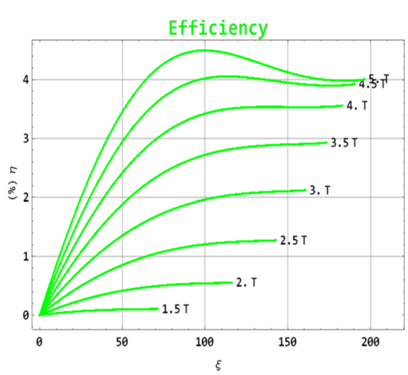
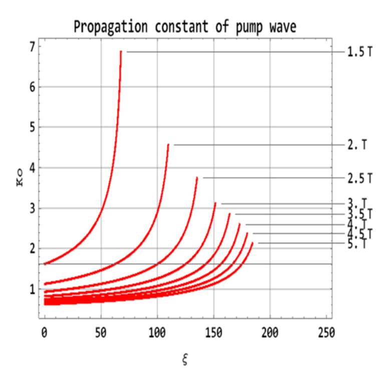
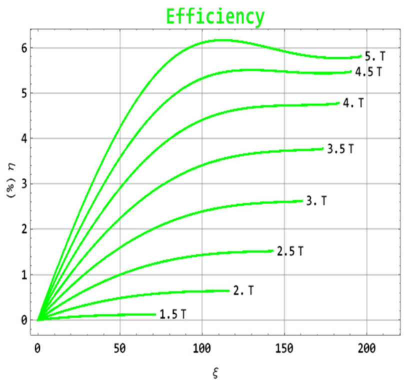
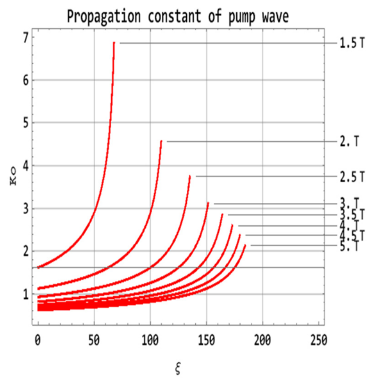
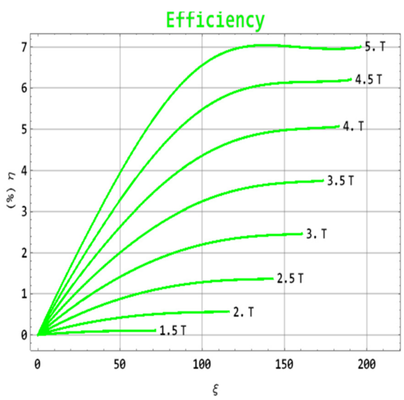
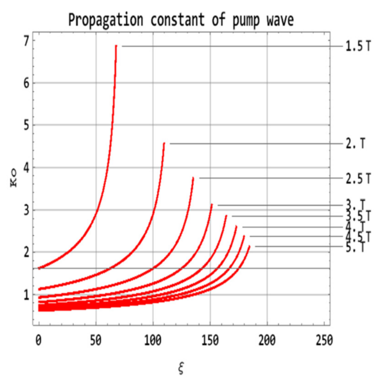
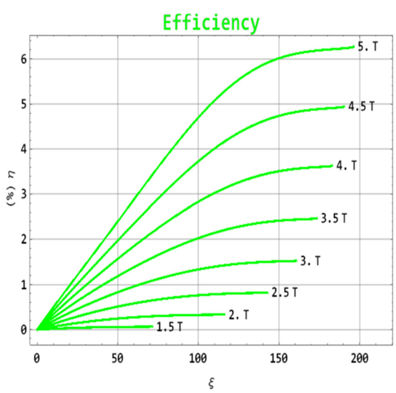
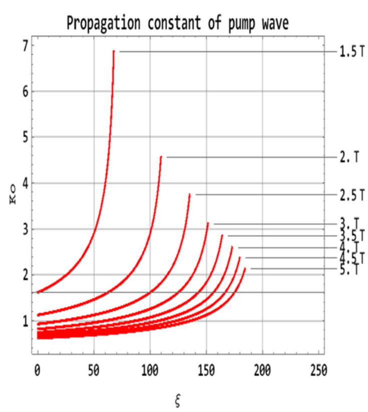
Injection angle of Electron beam	*Efficiency versus normalized length of FEL system for plasma density* $\boldsymbol{n_0}$ = $\mathbf{10^{20} m^{-3}}$, $\boldsymbol{\gamma}$ = **10**, $\boldsymbol{f_o}$ = **30GHz**, $\boldsymbol{a_1}$ = **0.0008**, a_0 = 0.13 *for different tapered static magnetic field*	*Normalized propagation constant versus normalized length of FEL system for plasma density* $\boldsymbol{n_0}$ = $\mathbf{10^{20} m^{-3}}$, $\boldsymbol{\gamma}$ = **10**, $\boldsymbol{f_o}$ = **30GHz**, $\boldsymbol{a_1}$ = **0.0008**, a_0 = 0.13 *for different tapered static magnetic field*
$\boldsymbol{\psi}$ = **0°**		
$\boldsymbol{\psi}$ = **30°**		
$\boldsymbol{\psi}$ = **45°**		
$\boldsymbol{\psi}$ = **60°**		

TABLE I. (*Continued.*)

Injection angle of Electron beam	*Efficiency versus normalized length of FEL system for plasma density* $\boldsymbol{n_0} = \mathbf{10^{20}m^{-3}}$, $\boldsymbol{\gamma} = \mathbf{10}$, $\boldsymbol{f_o} = \mathbf{30GHz}$, $\boldsymbol{a_1} = \mathbf{0.0008}$, $a_0 = 0.13$ *for different tapered static magnetic field*	*Normalized propagation constant versus normalized length of FEL system for plasma density* $\boldsymbol{n_0} = \mathbf{10^{20}m^{-3}}$, $\boldsymbol{\gamma} = \mathbf{10}$, $\boldsymbol{f_o} = \mathbf{30GHz}$, $\boldsymbol{a_1} = \mathbf{0.0008}$, $a_0 = 0.13$ *for different tapered static magnetic field*
$\boldsymbol{\psi} = \mathbf{90}°$		
$\boldsymbol{\psi} = \mathbf{120}°$		
$\boldsymbol{\psi} = \mathbf{150}°$		

The $z-$ component of beam momentum equation under the beat ponderomotive force can be written as

$$\frac{dP_{zb}}{dt} = v_{zb}\frac{dP_{zb}}{dz} = mc^2\frac{d\gamma}{dz} = F_{pbz},$$

or

$$\frac{d\gamma}{dz} = A\sin\psi,$$

where

$$\gamma = (1 - v_{zb}^2/c^2)^{-1/2},$$

$$A = -\frac{e^2 A_o^* A_1}{m^2 c^2 \omega_o \omega_1 \gamma_o^o}\left(k_o + \frac{(\omega_o + k_o v_{ob}^o)k_1}{\omega_o + k_o v_{ob}^o - \omega_c/\gamma_o^o}\right) \tag{9}$$

The equation governing ψ is

$$\frac{d\psi}{dz} = \frac{\omega}{v_{zb}} - (k_o + k_1),$$

or

$$\frac{d\psi}{dz} = \frac{\omega\gamma}{c(\gamma^2 - 1)^{1/2}} - (k_o + k_1) \tag{10}$$

After normalizing the Equation (9) and (10), one gets coupled equation for FEL as

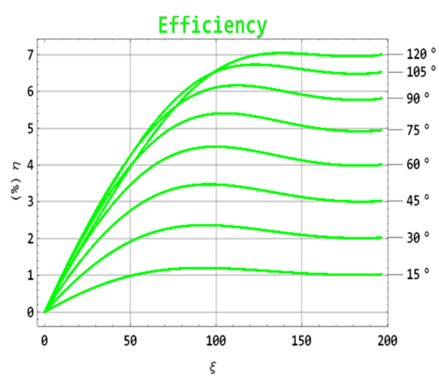
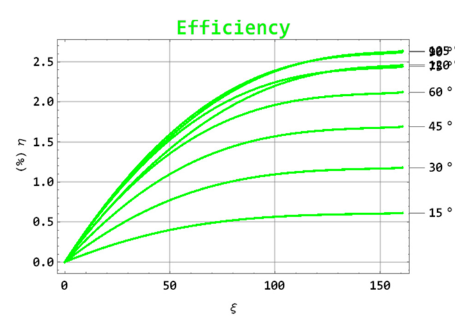
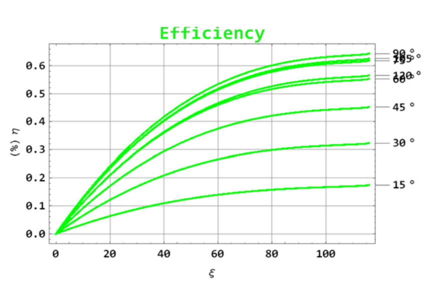
$$\begin{aligned}\frac{d\gamma}{d\xi} &= AL\sin\psi,\\ \frac{d\psi}{d\xi} &= \frac{\omega\gamma L}{c(\gamma^2-1)^{1/2}} - L(k_o + k_1)\end{aligned} \tag{11}$$

Where, $\xi = \frac{z}{L}$, L is the wavelength of electromagnetic wiggler wave.

Let the value of γ at $\xi = 0$ and $\xi = \frac{z}{L}$ be denoted as $\gamma(0)$ and $\gamma(\xi)$ respectively, then the net energy radiated by the trapped electrons is,

$$\Delta E = mc^2(\gamma(0) - \gamma(\xi)) \tag{12}$$

TABLE II. Comparison of Efficiency of FEL generation by varying tapered static magnetic field for different injection angle ψ of Electron Beam.

Tapered Static Magnetic field	*Efficiency versus normalized length of FEL system for plasma density* $\boldsymbol{n_0 = 10^{20} m^{-3}}$, $\boldsymbol{\gamma = 10}$, $\boldsymbol{f_o = 30GHz}$, $\boldsymbol{a_1 = 0.0008}$, $a_0 = 0.13$ *for different injection angle* ψ *of Electron Beam*
$B_{os} = 5$ T	
$B_{os} = 3$ T	
$B_{os} = 2$ T	

Now the efficiency of radiation of trapped electrons is given by,

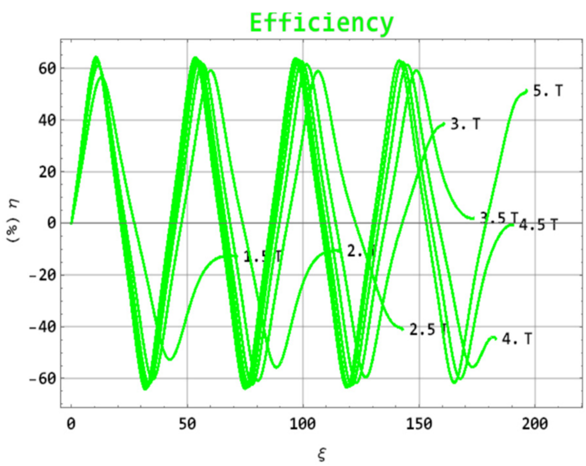
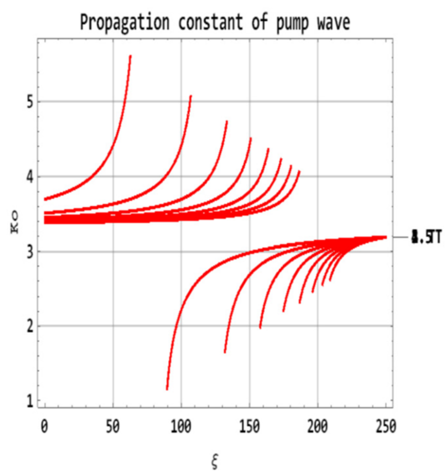
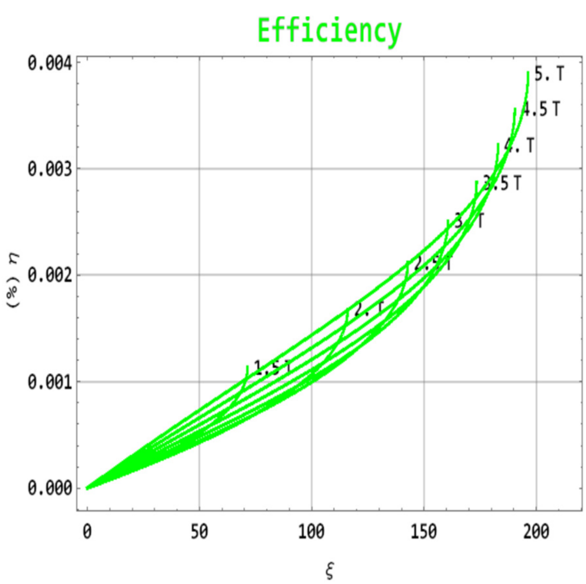
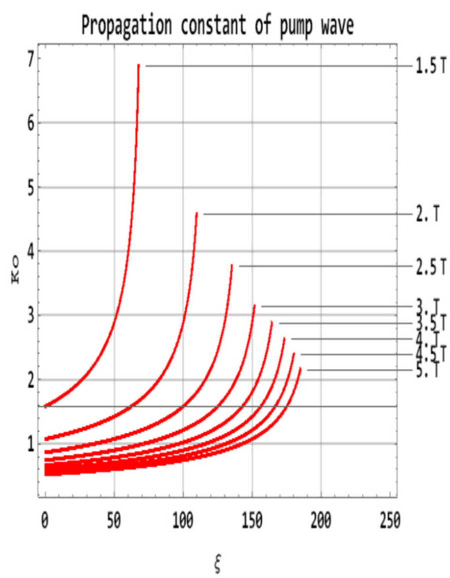
$$\eta = \frac{\Delta E}{mc^2(\gamma(0) - 1)} \tag{13}$$

To this if we multiply the fraction of electrons trapped, would get the overall efficiency of FEL radiation. In case, where the electron enter the interaction region $\xi = 0$ at an uniform rate i.e. they are distributed over ψ, then the fraction of those trapped electrons is $\frac{2\Delta\psi}{2\pi}$ or $\frac{\Delta\psi}{\pi}$, where $\Delta\psi = \psi(\xi) - \psi(0)$ is the net change in due to phase spread so the overall efficiency of the FEL radiation is given by,

$$\eta = \left(\frac{\gamma(0) - \langle\gamma(\xi = L_w/L)\rangle}{\gamma(0) - 1}\right)\left(\frac{\psi(\xi = L_w/L) - \psi(0)}{\pi}\right) \tag{14}$$

Where L_w is the length of wiggler and corresponds to the position of maximum gain. Here $\langle\gamma(\xi = L_w/L)\rangle$ represents average over N discrete electrons injected with the same $\gamma(\xi = 0)$ but with initial phase distributed uniformly between 0 to 2π.

TABLE III. Efficiency and propagation constant of FEL generation in a tapered static magnetic field by varying plasma density.

plasma density	*Efficiency versus normalized length of FEL system for* $\boldsymbol{\gamma} = \mathbf{10}$, $\boldsymbol{f_o} = \mathbf{30GHz}$, $\boldsymbol{a_1} = \mathbf{0.0008}$, $a_0 = 0.13$ *and* $\boldsymbol{\psi} = \mathbf{120}^\circ$ *for different tapered static magnetic field*	*Normalized propagation constant versus normalized length of FEL system for* $\boldsymbol{\gamma} = \mathbf{10}$, $\boldsymbol{f_o} = \mathbf{30GHz}$, $\boldsymbol{a_1} = \mathbf{0.0008}$, $a_0 = 0.13$ *and* $\boldsymbol{\psi} = \mathbf{120}^\circ$ *for different tapered static magnetic field*
$\boldsymbol{n_0} = \mathbf{10^{18}}\boldsymbol{m^{-3}}$		
$\boldsymbol{n_0} = \mathbf{10^{22}}\boldsymbol{m^{-3}}$		

III. RESULTS AND DISCUSSION

To generate the FEL in THz frequency range we propose to use gyrotron powered electromagnetic wave wiggler with power density ~ 2 GW/cm^2 at 30 GHz. For significant gain the beam current density ~ 10^9 A/m^2 and beam energy ~ 5MeV are required. In Table I, plot of efficiency of FEL and propagation constant of whistler pump wave as a function of normalized length of the FEL at various injection angles of electron beam ($\boldsymbol{\psi}$ = **0**°, **30**°, **60**°, **90**°, **120**°, **150**°) on them at plasma density $\boldsymbol{n_0 = 10^{20} m^{-3}}$, $\boldsymbol{\gamma = 10}$, $\boldsymbol{f_o}$ = **30GHz**, $\boldsymbol{a_1}$ = **0.0008**, $a_0 = 0.13$ for different tapered static magnetic fields B_{os} = (1.5T – 5.0T) are compared. Here ψ is phase of the beam electrons, so different phase angle of electron beams basically reflects the different injection angle of electron beams which can be controlled. From Table I, with varying the injection angle of electron beam the impact on the change in the efficiency can be observed.

For B_{os} = 1.5T propagation constant of wiggler goes on increasing for $L_w \sim 75L$ and lasing is stopped. Increasing of the magnetic field causes increase in lasing action for $L_w > 75L$ yielding higher efficiency of FEL but the useful L_w is between 75L – 100L for $\boldsymbol{\psi}$ = **0**° and $\boldsymbol{\psi}$ = **30**°. For increasing $\boldsymbol{\psi}$ higher efficiency of FEL is yielded for higher L_w. For $\boldsymbol{\psi}$ = **120**°, B_{os} = 5T at $L_w \sim 120L$ efficiency of 7% of FEL is obtained. Hence to yield higher efficiency of FEL $\boldsymbol{\psi}$ has to be adjusted between **90**° and **150**° as efficiency starts dropping after **120**°.

In Table II, plot of efficiency of FEL as a function of normalized length of the FEL for different tapered static magnetic fields (B_{os} = 2T, 3T, 5T) at plasma density $n_0 = 10^{20} m^{-3}$, $\gamma = 10$, f_o = 30GHz, a_1 = 0.0008, a_0 = 0.13 at various injection angles of electron beam (ψ = 0°, 30°, 60°, 90°, 120°, 150°) are compared. We observe for B_{os} = 5T at $L_w = 100L$ the FEL efficiency is equal for ψ = 105° and ψ = 120°, i.e. crossover for ψ = 120° of FEL efficiency is occurring as for ψ = 105° for wiggler length $L_w > 100L$, FEL efficiency drops. Also, we observe that the injection angle of electron beam is the important factor for higher efficiency of FEL as efficiency is increasing for increasing ψ for constant tapered magnetic field but the higher wiggler length L_w is required. These results are itself an indication of the importance of tapering of magnetic field to extract more energy from electron beam to FEL and hence higher wiggler length is required.

From Table III, plot of efficiency of FEL and propagation constant of whistler pump wave as a function of normalized length of the FEL for different plasma density ($n_0 = 10^{18} m^{-3}$, $10^{22} m^{-3}$), $\gamma = 10$, f_o = 30GHz, a_1 = 0.0008, a_0 = 0.13 at different tapered static magnetic fields (B_{os} = 1.5T, 2T, 2.5T, 3T, 3.5T, 4T, 4.5T, 5T)are compared. Also, the figure for efficiency of FEL from Table I (Row7, Column 2) for ψ = 120° can also be compared with the figures in Table III as this figure is for $n_0 = 10^{20} m^{-3}$. The beam density of value $n_{0b} \sim 10^{21} m^{-3}$ for beam current density ~ 10^9 A/m^2 and beam energy ~ 5MeV is required and hence we observe that for plasma density $n_0 = 10^{18} m^{-3}$ lower than the beam density the FEL efficiency of 60% at $L_w \sim 13L$ is achieved but is returned back to the system at $L_w \sim 25L$ i.e. oscillatory in nature and for $n_0 = 10^{22} m^{-3}$ efficiency is much lower. From Table III, it is observed that the impact of plasma density variation is more pronounced on the efficiency of FEL.

It is clear by the resulting figures shown in all tables that by employing the varying magnetised plasma medium to the interaction regime of a FEL, the wavelength of electromagnetic wiggler wavelength can be significantly reduced. Both the electron plasma frequency and electron cyclotron frequency affect dispersion relation of the wiggler wave. This simulation results indicates that tapering of magnetic field can improve the FEL efficiency by the suitable choice of plasma density and magnetic field strength. For the wiggler of 30 GHz frequency, we can generate 6 THz frequency as $\omega_1 \sim 2\gamma_o^2 \omega_o$. Power gain and FEL efficiency are improved by tapering, plasma and controlled injection angle of electron beam.

ACKNOWLEDGMENTS

The author would like to thank Prof. V. K. Tripathi for his valuable guidance and analytical formulation and Prof. Jitendra Kumar for his important observations in the graphs. The author would also like to acknowledge the reviewer whose constructive comments had helped in improving the paper.

REFERENCES

[1] M. Kumar, L. Bhasin, and V. K. Tripathi, Phys. Scr. **81**, 045504 (5pp) (2010).

[2] F. Lehmkuhler, C. Gutt, B. Fischer, M. A. Schroer, M. Sikorski, S. Song, W. Roseker, J. Glownia, M. Chollet, S. Nelson, K. Tono, T. Katayama, M. Yabashi, T. Ishikawa, A. Robert, and G. Grubel, "Single shot coherence properties of the free-electron laser SACLA in the hard x-ray regime," Scientific Reports (2014).

[3] M. Vogt, B. Faatz, J. Feldhaus, K. Honkavaara, S. Schreiber, and R. Treusch, "The free-electron laser FLASH at DESY," Proceedings of IPAC2013, Shanghai, China, TUPEA004, (2013).

[4] H. Yoshida, N. D. Munemoto, T. Kimura, T. Sano, S. Yamaguchi, and M. R. Asakawa, "Analysis of the double-slab type Cherenkov free-electron laser resonator," Translated from Denki Gakkai Ronbunshi, Vol. 132-A(9), September 2012, pp. 960–967 and Electronics and Communications in Japan **97**(6) (2014).

[5] A. Hasanbeigi, A. Farhadian, and E. K. Bidhendi, "Growth rate enhancement of free-electron laser by two consecutive wigglers with axial magnetic field," Indian Academy of Sciences **82**(6), 1049–1060 (2014).

[6] S. H. Gold, W. M. Black, H. P. Freund, V. L. Granatstein, and A. K. Kinkead, "Radiation growth in a millimeter-wave free electron laser operating in the collective regime," Physics of Fluids **27**(3), 746–754 (1984).

[7] S. H. Gold, H. P. Freund, and Bowie, "Free electron laser with tapered axial magnetic field," The United States of America as represented by the Secretary of the Navy, Washington, DC, 1987, Patent Number: 4,644,548.

[8] T. J. Orzechowski, B. R. Anderson, W. M. Fawley, D. Prosnitz, E. T. Scharlemann, and S. M. Yarema, "High gain and high extraction efficiency from a free electron laser amplifier operating in the millimeter wave regime," Nuclear Instruments and Methods in Physics Research A **250**, 144–149 (1986).

[9] A. Sharma and V. K. Tripathi, "A whistler pumped free electron laser," Phys. Fluids **3I**, 3375–3378 (1988).

[10] A. Sharma and V. K. Tripathi, "A plasma filled gyrotron pumped free electron laser," Phys. Plasmas **3**(8), 3116–3120 (1996).

[11] K. K. Pant and V. K. Tripathi, "Nonlocal theory of a whistler pumped free electron laser," Physics of Plasmas **1**, 1025 (1994), IRACST – Engineering Science and Technology: An International Journal (ESTIJ), ISSN: 2250-3498 Vol.8, No.2, Mar-April 2018 Page 10.

[12] K. K. Pant and V. K. Tripathi, "Free electron laser operation in the whistler mode," IEEE Transactions on Plasma Science **22**(3) (1994).